

We have studied the spectra of temperature fluctuations and the heat-transfer coefficient in a disperse medium, and have compared the results with experimental data.

Using the Glensdorf-Prigozhine local potential method [1], Vasanova et al. [2] treated nonequilibrium temperature fluctuations relative to a stationary distribution of the mean temperature of a fluidized bed and derived an expression for the time correlation of the fluctuation function which was confirmed by experiment.

In the present article we use the spectral theory of random processes to investigate the frequency spectrum of such fluctuations independently. Since temperature fluctuations are generated by a mechanism which is closely related to the disturbance of the uniformity of the bed structure, we first consider fluctuations of the volume density of the particles. According to [1] the probability of macroscopic fluctuations in nonequilibrium systems can be calculated with the Einstein formula $W \sim \exp(-\Delta S)$. The incremental deviation ΔS of the configurational entropy from its maximum value can be calculated by statistical physics methods developed in [3] for a fluidized bed. The fluctuating motion of particles in a fluidized bed is described by the Gibbs canonical distribution [3, 4]. This enables one to evaluate the phase integral

$$Z = \frac{1}{N!} \int_{\{\Gamma\}} \exp(-H/\Theta) d\Gamma = \frac{1}{N!} V_f^N (2\pi m\Theta)^{3/2N}. \quad (1)$$

In contrast with [3], we define the free volume V_f as the difference between the settled out volume V of the bed and the volume V_{1*} occupied by the particles under close packing [5]. Per particle we have

$$\sigma_f = \frac{V_f}{N} = \frac{1}{N} [(V - V_{1*}) = \sigma - \sigma_* = V_p \rho_N^{-1} \left(1 - \frac{\rho_N}{\rho_*}\right), \quad (2)$$

$$\sigma = \frac{V}{N} = V_p \rho_N^{-1} = n^{-1}, \quad \sigma_* = \frac{V_{1*}}{N} = V_p \rho_*^{-1}.$$

The effective free energy is

$$F = -\Theta \ln Z = -\Theta N \left[\ln \frac{V_f}{N} + \frac{3}{2} \ln(2\pi m\Theta) \right] + N\Theta. \quad (3)$$

Hence the specific configurational entropy is

$$s = \frac{1}{V} \left(\frac{\partial F}{\partial \Theta} \right)_V = n \ln \left[n \left(1 - \frac{\rho_N}{\rho_*}\right) \right] + s_\Theta. \quad (4)$$

We consider approximate "coarse-grained" fluctuations $\delta\rho_N$ for a certain volume of the bed [3]. To terms of the order $(\delta\rho_N)^2$ the deviation of the total entropy of the system from the equilibrium value is

$$\Delta s = s - \langle s \rangle \approx \delta s + \frac{1}{2} \delta^2 s = -\frac{n}{2} (\delta\rho_N)^2 \left[\langle \rho_N \rangle^2 \left(1 - \frac{\langle \rho_N \rangle}{\langle \rho_* \rangle}\right) \right]^{-1},$$

$$\delta\rho_N = \frac{1}{V} \int \varphi dV, \quad (\delta\rho_N)^2 = \frac{1}{V} \int \varphi^2 dV, \quad \varphi = \rho_N - \langle \rho_N \rangle. \quad (5)$$

S. M. Kirov Ural Polytechnic Institute, Sverdlovsk. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 47, No. 1, pp. 116-124, July, 1984. Original article submitted February 14, 1983.

According to the Einstein formula the probability of fluctuation has the form

$$\mathbb{W}(\delta\rho_N) = \frac{n}{2\pi} \left[\langle \rho_N \rangle^2 \left(1 - \frac{\langle \rho_N \rangle}{\rho_*} \right) \right]^{-1} \exp \left\{ -\frac{n}{2} (\delta\rho_N)^2 \left[\langle \rho_N \rangle^2 \left(1 - \frac{\langle \rho_N \rangle}{\rho_*} \right) \right]^{-1} \right\}. \quad (6)$$

From (5) and (6) there follows an expression for the mean square of the fluctuations of the volume concentration of the particles

$$\langle (\delta\rho_N)^2 \rangle = \langle \rho_N \rangle^2 \left(1 - \frac{\langle \rho_N \rangle}{\rho_*} \right). \quad (7)$$

We note that this same result can be obtained by generalizing Smolukovskii's combinatorial method [6, 7].

In the analysis of the temperature fluctuations we assume, as in [2], that δT obeys the hyperbolic heat-conducting equation

$$\tau_T \frac{\partial^2 \delta T}{\partial t^2} + \frac{\partial \delta T}{\partial t} = a_{ik} \frac{\partial}{\partial x_i} \frac{\partial \delta T}{\partial x_k}. \quad (8)$$

The connection between temperature fluctuations and hydrodynamic fluctuations requires considering the complete system of hydrodynamic and heat-transfer equations, which greatly complicates the problem even for a single-phase medium [8]. An approximate solution can be obtained by using Landau's idea [9] of introducing external sources of fluctuations into the dynamic equations. This approach was used in the analysis of fluctuations of the concentration of solid particles of a disperse system [7]. We shall follow this method, considering temperature instead of concentration.

We introduce a random source $y_T(\vec{r}, t)$ into the right-hand side of (8), and express δT and y_T in terms of Fourier-Stieltjes stochastic integrals

$$\delta T = \int \exp[i(\vec{k} \cdot \vec{r} + \omega t)] dZ_T, \quad y_T = \int \exp[i(\vec{k} \cdot \vec{r} + \omega t)] dZ_y. \quad (9)$$

Substituting (9) into (8), we obtain an equation relating the spectral measures dZ_T and dZ_y :

$$(i\omega - \tau_T \omega^2 + a_{ik} k_i k_k) dZ_T = dZ_y. \quad (10)$$

Since within the framework of the present model the temperature is considered as a passive transportable admixture, local temperature fluctuations δT must be first of all related to fluctuations $\delta\rho_N$ of the volume concentration of the solid phase, with the dynamics described by the hyperbolic diffusion equation [7]. In order to take account of the buildup of fluctuations compensating their damping, we introduce a random source y_ρ into the diffusion equation. The characteristic time of change of y_ρ is of the same order of magnitude as the time of action of the hydrodynamic forces which produce the fluctuation, i.e. of the same order as the inner scale of pseudoturbulence τ .

Since the lifetime of the fluctuations $\tau_\rho \gg \tau$, their random buildup can be considered as a process with independent increments, which leads to a spectral density of the source $\Psi_{\rho\rho} = \Psi_{\rho\rho}(\vec{k})$ which is independent of frequency. Taking account of the above, we make the same assumption with respect to the source y_T in the heat-conduction equation. Multiplying (9) by the complex conjugate and taking the statistical average, we obtain an equation for the spectral density of the random process $\delta T(\vec{r}, t)$:

$$\Psi_{TT}(\omega, \vec{k}) = \frac{\Psi_{yy}(\vec{k})}{\omega^2 + (a_{ik} k_i k_k - \tau_T \omega^2)^2}, \quad \Psi_{yy}(\vec{k}) = \lim_{d\omega, d\vec{k} \rightarrow 0} \frac{\langle dZ_y^* dZ_y \rangle}{d\omega d\vec{k}}. \quad (11)$$

Integrating (11) with respect to the frequency, we obtain

$$\Phi_{TT}(\vec{k}) = \Psi_{yy}(\vec{k}) \int_{-\infty}^{+\infty} \frac{d\omega}{\omega^2 + (a_{ik} k_i k_k - \tau_T \omega^2)^2} \simeq \frac{\pi}{a_i k_i k_k} \Psi_{yy}(\vec{k}). \quad (12)$$

Thus

$$\Psi_{TT}(\omega, \vec{k}) = \frac{1}{\pi} \frac{a_{ik} k_i k_k \Phi_{TT}(k)}{\omega^2 + (a_{ii} k_i k_i - \tau_T \omega^2)^2}. \quad (13)$$

The spectral density of the random process $\delta\rho_N(\vec{r}, t)$ is described by an analogous relation in which the diffusion coefficient tensor D_{ik} enters instead of a_{ik} [7]. Since at present we know of no experimental data on the spatial correlations of temperature fluctuations in disperse systems, we simplify further calculations by restricting ourselves to a one-dimensional approximation. In order to find the relation between the spectral densities of the random processes $\delta T(\vec{r}, t)$ and $\delta\rho_N(\vec{r}, t)$, we consider the temperature fluctuations of a single particle δT_1 . Assuming that the stationary distribution of the mean temperature in the region occupied by the fluctuations is linear, using the relaxation approximation

$$\frac{d}{dt} \delta T_1 = \frac{1}{\tau_T} (\delta T_1 - E x), \quad (14)$$

where $\tau_T = c_1 \rho_1 R / 3\alpha$, and differentiating (14) with respect to time, we obtain the dynamic equation for δT_1 :

$$\tau_T \frac{d^2}{dt^2} \delta T_1 - \frac{d}{dt} \delta T_1 + E \delta v_1 = 0. \quad (15)$$

Here $\delta v_1 = dx/dt$ is the fluctuating velocity of the particle, and x is its random Lagrangian coordinate. Expressing δT_1 and $x(t)$ in (15) in terms of Fourier-Stieltjes stochastic integrals, we find

$$(\tau_T \omega^2 + i\omega) dZ_{T_1} = i\omega dZ_x. \quad (16)$$

Similarly, from the relation $T = (1 + i\omega\tau_T)T_1$, which follows from Newton's law for convective heat transfer, we obtain

$$dZ_T = (1 + i\omega\tau_T) dZ_{T_1}. \quad (17)$$

By using Eqs. (16) and (17) we find the spectral densities of the random processes δT_1 and δT in the standard way:

$$\Psi_{T_1 T_1} = E^2 (1 + \tau_T \omega^2) \Psi_{xx}, \quad \Psi_{TT} = (1 + \omega^2 \tau_T^2) \Psi_{T_1 T_1}, \quad \Psi_{TT}(\omega, \vec{k}) = E^2 \Psi_{xx}. \quad (18)$$

The determination of the relations between the Lagrangian and Eulerian characteristics of turbulent flows is a complicated problem of statistical fluid mechanics [8], and attempts to solve it theoretically have so far not given any practical results.

Random displacements of particles can be expressed in general form in terms of fluctuations of their volume concentration $\delta\rho_N$ by using the equation of continuity in Lagrangian form $\rho_N J = \text{const}$. In the linear approximation we have

$$\delta x = -A \delta\rho_N, \quad A = J \left[\rho_N \left(\frac{\partial J}{\partial x} \right) \right]^{-1}. \quad (19)$$

This gives a relation between the spectral densities:

$$\Psi_{xx} = A^2 \Psi_{\rho\rho}, \quad \Psi_{TT} = (AE)^2 \Psi_{\rho\rho}, \quad \Phi_{TT}(k) = (AE)^2 \Phi_{\rho\rho}(k). \quad (20)$$

In spite of the fact that an explicit form of the particle spectral function $\Phi_{\rho\rho}(k)$ was found in [5, 7], Eq. (20) does not settle the question of the spectral density of the correlation of the temperature fluctuations, since A must be determined independently, and this requires the use of supplementary considerations based, for example, on experimental data or on hypotheses analogous to those in the spectral theory of turbulence of single-phase media [8] and in statistical fluid mechanics of disperse systems [5, 10].

Using Eqs. (6) and (7), we can write $\Phi_{\rho\rho}(k)$ in the form [10]

$$\Phi_{\rho\rho}(k) = \frac{\pi}{k_m} \langle \delta\rho_N^2 \rangle \exp \left[- \left(\frac{k_m - k}{k_m} \right)^2 \right], \quad (21)$$

where k_m is the wave number at which the short-wave part of the spectrum is broken off [6, 7]. For sufficiently large k_m the asymptotic form of (21) can be used:

$$\Phi_{\rho\rho}(k) \simeq \langle \delta\rho_N^2 \rangle \delta(k_m - k). \quad (22)$$

Substituting (22) into (20), and then (20) into (13), assuming that the density Ψ_{TT} is one-dimensional, and integrating with respect to k , we find the frequency spectrum of the temperature fluctuations at a fixed point of a disperse medium

$$\bar{S}_{TT}(\omega) = \frac{S_{TT}(\omega)}{\langle \delta T^2 \rangle} = \frac{\omega_0^2}{\pi \tau_r \left[(\omega^2 - \omega_0^2)^2 + \left(\frac{\omega}{\tau_r} \right)^2 \right]}, \quad (23)$$

$$\langle \delta T^2 \rangle = (AE)^2 \langle \delta \rho_N \rangle, \quad \omega_0^2 = k_w a / \tau_r.$$

Applying the Wiener-Khinchin theorem to this spectrum, we obtain the correlation function in the form [2]

$$\varphi_{TT}(t) = \langle \delta T^2 \rangle \exp\left(-\frac{|t|}{2\tau_r}\right) \left(\cos \bar{\omega} t + \frac{1}{2\omega\tau_r} \sin \bar{\omega} |t| \right), \quad (24)$$

$$\bar{\omega} = \sqrt{\frac{k_m a}{\tau_r} - \frac{1}{4\tau_r^2}} = \sqrt{\omega_0 - \frac{1}{(2\tau_r)^2}}.$$

We also consider thermal fluctuations of the body which exchanges heat with the fluidized bed. For a random temperature distribution in the bed we use (8). In doing this we assume that the distribution of mean temperatures obeys the steady-state heat-conduction equation. These conditions can be ensured, for example, by sufficiently intense internal heat sources in the body. In particular, these conditions hold in studying heat transfer in a bed with plate thermoanemometers [10-13].

In Eq. (8) for temperature fluctuations it is convenient to go over to the Fourier components $\hat{\delta T}$. For a one-dimensional problem we have:

$$(i\omega - \tau_r \omega^2) \hat{\delta T}(\omega, x) = -\frac{\lambda}{c'} \frac{d^2}{dx^2} \hat{\delta T}(\omega, x), \quad \hat{\delta T}(\omega, 0) = \hat{\delta T}_s. \quad (25)$$

Since in using the random source method [9] we are investigating quasistationary (smoothed) fluctuations corresponding to the low-frequency part of the spectrum, we can limit ourselves to an approximate solution of (25). This can be obtained, for example, by the integral heat balance method [14]. Integrating both sides of (25) with respect to x from 0 to the thickness l of the thermal layer, and neglecting in this approximation the relaxation terms, we obtain

$$i\omega c' \int_0^l \hat{\delta T}(\omega, x) dx = -\lambda \left(\frac{d\hat{\delta T}}{dx} \right)_{x=0} = \delta q_s, \quad (26)$$

$$\hat{\delta T}(\omega, l) = 0, \quad \left(\frac{d\hat{\delta T}}{dx} \right)_{x=l} = 0.$$

Using the conditions on the boundaries of the thermal layer and the heat balance for the body under consideration, we can write

$$\hat{\delta T} = \hat{\delta T}_s \left(1 - \frac{x}{l} \right)^2, \quad K \frac{\partial \delta T}{\partial t} = -\delta q_s, \quad i\omega K \hat{\delta T}_s = -\hat{\delta q}_s. \quad (27)$$

Here K plays the role of the effective heat reception, and for small Biot numbers $K = Vc'_s/F$. It follows from (26) and (27) that $l = 3K/c'$. Using (25) for the heat-transfer surface

$$(i\omega - \tau_r \omega^2) \hat{\delta T}_s = \frac{\lambda}{c'} \left(\frac{d^2 \hat{\delta T}_s}{dx^2} \right)_{x=0} + \hat{y}_s, \quad (28)$$

where \hat{y}_s is the Fourier component of the source of random fluctuations, and evaluating the derivative on the right-hand side by using the distribution in (27), we find

$$\left(i\omega - \tau_r \omega^2 + \frac{2}{9} \frac{\lambda c'}{K^2} \right) \hat{\delta T}_s = \hat{y}_s. \quad (29)$$

Multiplying the polynomial (29) by the complex conjugate, we obtain the required spectral density

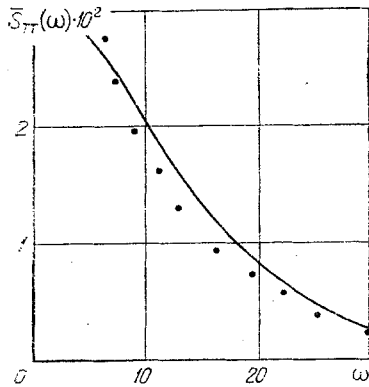


Fig. 1

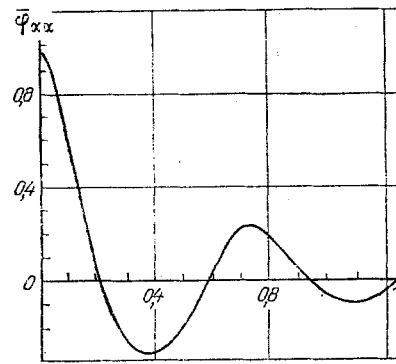


Fig. 2

Fig. 1. Spectral density of temperature fluctuations in a fluidized bed; points are from experiments [2, 18]. $\bar{S}_{TT}(\omega) \cdot 10^2$ is in sec, and ω is in sec^{-1} .

Fig. 2. Autocorrelation function of fluctuations of heat-transfer coefficient in a fluidized bed. τ is in sec.

$$S_{T_s T_s}(\omega) = \frac{1}{\pi} \frac{\langle \delta T_s^2 \rangle \omega_0^2}{\tau_r \left[(\omega^2 - \omega_0^2)^2 + \left(\frac{\omega}{\tau_r} \right)^2 \right]}, \quad (30)$$

$$\hat{y}_s \hat{y}_s^* = \frac{2}{\pi} \frac{\lambda c'}{9K^2}, \quad \omega_0^2 = \frac{2}{9} \frac{\lambda c'}{K^2 \tau_r}.$$

Applying the Wiener-Khinchin theorem to (30), we obtain the correlation function of the surface temperature fluctuations in the form (24), where in this case $\bar{\omega} = \sqrt{2\lambda c' / 9K^2 \tau_r - 1/4\tau_r^2}$. We find the spectral density of the fluctuations of the heat-transfer coefficient by using the heat balance (27) of the body under consideration and the relations

$$\alpha = \frac{q_s}{\Delta T} = \frac{\langle q_s \rangle + \delta q_s}{\langle \Delta T \rangle + \delta T_s}, \quad \langle q_s \rangle = \langle \alpha \rangle \langle \Delta T \rangle, \quad (31)$$

$$\alpha = \langle \alpha \rangle + \delta \alpha.$$

From (27) and (31) we find a relation between the Fourier components of the fluctuations of the heat-transfer coefficient $\delta \hat{\alpha}$ and the surface temperature $\delta \hat{T}_s$:

$$\delta \hat{\alpha} = - \frac{\langle \alpha \rangle}{\langle \Delta T \rangle} \left(\frac{i\omega K}{\langle \alpha \rangle} + 1 \right) \delta \hat{T}_s. \quad (32)$$

Using (30) and (32), we find

$$S_{\alpha\alpha}(\omega) = \frac{\langle \delta \alpha^2(0) \rangle \left(\omega^2 + \frac{\langle \alpha^2 \rangle}{K^2} \right)}{2\pi \tau_r \left[\left(\omega^2 - \frac{2}{9} \frac{\lambda c'}{K^2 \tau_r} \right)^2 + \left(\frac{\omega}{\tau_r} \right)^2 \right]}, \quad (33)$$

where

$$\langle \delta \alpha^2 \rangle = \frac{4}{9} \frac{\lambda c' \langle \delta T_s^2 \rangle}{\langle \Delta T \rangle^2 \tau_r}.$$

The corresponding correlation function has the form

$$\varphi_{\alpha\alpha} = \frac{\varphi_{\alpha\alpha}(t)}{\langle \delta \alpha^2 \rangle} = \exp \left(\frac{|t|}{2\tau_r} \right) \cos \bar{\omega} t. \quad (34)$$

The last relation holds if the following equation for the average heat-transfer coefficient is satisfied:

$$\langle \alpha \rangle \simeq \sqrt{\frac{2}{9} \frac{\lambda c'}{\tau_r}} \quad (35)$$

To within a numerical factor Eq. (35) agrees with the well-known Mickley-Fairbanks formula [11, 13]. In the latter $1/\pi$ appears instead of $2/9$, which for low-frequency fluctuations agrees with experiment. Equation (35) was generalized for high frequencies within the framework of the "packet" model of heat transfer in [11]. It should be noted that formula (35) was obtained by using the hyperbolic equation (8) in which the relaxation time τ_r has to be determined independently. In principle it is possible to characterize the relaxation of the heat capacity, the thermal conductivity, the heat flux, or the temperature gradient. While the data favor the use of the hyperbolic heat-conduction equation for modeling heat transfer in a fluidized bed [15], at present not all these questions have been completely cleared up [16, 17].

Figure 1 shows the spectral density of the temperature fluctuations in a fluidized bed [2, 18]. The following data from [2] were used in calculations with (23): $\bar{\omega} = 7.58 \text{ sec}^{-1}$, $\tau_r = 0.0286 \text{ sec}$. The value $\omega_0 = 19.1 \text{ sec}^{-1}$ was determined from (24). Using Eq. (23) for ω_p , the wave number at which the spatial spectrum of the fluctuations is broken off was estimated to be $k_m = \omega_0 \sqrt{\tau_r/a} = 304 \text{ m}^{-1}$. An estimate of this same quantity in terms of the free volume σ_f (2) gives $k_m \sim (\langle \rho_N \rangle^{1/3} / 2R) (1 - \langle \rho_N \rangle / \rho_*) = 360 \text{ m}^{-1}$. The values found for k_m are of the same order of magnitude as those calculated with the formulas given in [5, 10]. The thermal conductivity λ of the bed was calculated with the dimensionless equation given in [19]; the volumetric heat capacity $\rho c = \rho_0 \epsilon c_0 + (1 - \epsilon) \rho_1 c_1 = 4.05 \cdot 10^6 \text{ J/m}^3 \cdot \text{K}$ ($c_0 = 4.19 \text{ kJ/kg} \cdot \text{K}$, $c_1 = 0.8 \text{ kJ/kg} \cdot \text{K}$, $\epsilon = 0.9$, $\rho_0 = 998 \text{ kg/m}^3$, $\rho_1 = 3590 \text{ kg/m}^3$). The thermal diffusivity of the bed $a = \lambda/c' = 458/4.05 \cdot 10^6 = 1.13 \cdot 10^{-4} \text{ m}^2/\text{sec}$.

Figure 2 shows the correlation function of the heat-transfer coefficient fluctuations obtained by using a plate thermoanemometer in a fluidized bed of corundum particles $270 \mu\text{m}$ in diameter fluidized by air with a filtration velocity of 0.27 m/sec (pistonlike conditions). The anemometer consisted of a $25 \times 5 \times 1.5 \text{ mm}$ plate of epoxy resin with $5\text{-}\mu\text{m}$ -thick platinum foil cemented to the flat sides. The curve in Fig. 2 is well approximated by function (34) with $\tau_r = 0.326 \text{ sec}$ and $\bar{\omega} = 7.85 \text{ sec}^{-1}$. The heat-transfer coefficient calculated with (35) is $\langle \alpha \rangle = 387 \text{ W/m}^2 \cdot \text{K}$ ($c' = 3.42 \cdot 10^2 \text{ kJ/kg} \cdot \text{K}$, $\lambda = 0.646 \text{ W/m}^2 \cdot \text{K}$ [11]) and its experimental value is $330 \text{ W/m}^2 \cdot \text{K}$. The heat flux δq^B in the thermoanemometer backing must be taken into account in the heat-balance equation (27). For harmonic temperature fluctuations $\delta T_B = \delta T(x) \exp(i\omega t)$ it follows from the solution of the heat-conduction equation that

$$\delta q_\omega^B = \frac{\lambda_B}{\sqrt{2} \omega} \sqrt{\frac{|\omega|}{a_B}} \frac{d\delta T_s}{dt} \quad (36)$$

Averaging (36) over the spectrum of temperature fluctuations $S_{T_s T_s}(\omega)$ (30), we obtain

$$\delta q^B = 2 \int_0^\infty \delta q^B S_{T_s T_s}(\omega) d\omega \simeq \frac{3}{4} \sqrt{\frac{\lambda_B \rho_B c_B}{2\omega_0^3 \tau_r^2}} \frac{d\delta T_s}{dt} \quad (37)$$

Using (37) it follows from the heat balance in (27) that

$$K \simeq K_F + \frac{3}{4} \sqrt{\frac{\lambda_B \rho_B c_B}{2\omega_0^3 \tau_r^2}} \quad (38)$$

For the anemometer used to obtain the curve in Fig. 2 ($\rho_B = 1.18 \cdot 10^3 \text{ kg/m}^3$, $c_B = 1.91 \text{ kJ/kg} \cdot \text{K}$, $\lambda_B = 0.2 \text{ W/m} \cdot \text{K}$, $\rho_F = 21.45 \cdot 10^3 \text{ kg/m}^3$, $c_F = 0.13 \text{ kJ/kg} \cdot \text{K}$, $\lambda_F = 70 \text{ W/m} \cdot \text{K}$) calculation with (38) gives $K = 50 \text{ J/m}^2 \cdot \text{K}$ for $\omega_0 = 8 \text{ sec}^{-1}$. An estimate of this same quantity based on experimental data for $\langle \alpha \rangle$ (35) and ω_0 (30) gives $K = \langle \alpha \rangle / \omega_0 = 330/8 = 41 \text{ J/m}^2 \cdot \text{K}$. Thus, the theoretical analysis of thermal fluctuations is in satisfactory agreement with the experimental data. In conclusion, we note that since (38) enters the expression for the spectral densities (30), (33), the anemometer because of its thermal inertia acts as an unusual frequency filter, cutting out the finest details of the spectrum.

NOTATION

W, probability; ΔS , change in entropy; Z, phase integral; H, Hamiltonian; θ , effective particle temperature; N, number of particles in system; m, mass of a particle; V, V_f , V_{1*} , volumes occupied by bed, gas, and close-packed particles, respectively; (σ , σ_f , σ_* , same per particle); ρ , density of material; ρ_N , ρ_* , volume density of particles in bed and under

close packing; F , free energy; n , particle concentration; t , running time; τ_r , relaxation time; α , thermal diffusivity; T , δT , temperature and its fluctuation; $\vec{r}(x, y, z)$, radius vector; \vec{k} , wave vector; ω , frequency; J , Jacobian; y , source of random fluctuations; Ψ , ϕ , S , spectral densities; φ , correlation function; R , particle radius; ω_0 , $\bar{\omega}$, characteristic frequencies determined in (23), (24), and (30), q , δq , heat flux and its fluctuations; λ , thermal conductivity; c , specific heat; c' , volumetric heat capacity; l , thickness of thermal layer; K , heat reception introduced in (27); α , heat-transfer coefficient; ϵ , porosity; $\langle \dots \rangle$, average. Subscripts: 0, continuous medium, 1, solid phase; s, surface; B, backing; F, foil.

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